

Guide to **Mathematics** Intervention Solutions:

A ROADMAP FOR STUDENT SUCCESS

Dr. Christy L. McGuire, Research Manager, Carnegie Learning, Inc.
Dr. Steve Ritter, Chief Product Architect, Carnegie Learning, Inc.

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I. Need for Interventions

Every school district in America has a population of students who either are at risk of failing, or have already failed. In some large urban districts, this population represents almost half of the students. An immediate and significant intervention is needed.

America's math problem is evident in the TIMSS (Mullis, et. al, 1998) and TIMSS-R (Mullis, et. al, 2000) studies and the 2004 PISA study (Programme for International Student Assessment; 2004) of math competency. In the 1995 TIMSS study, the U.S. ranked 18th of 21 countries in mathematics achievement at the end of high school. In the 1999 TIMSS study, the U.S. ranked 19th of 38 countries in eighth grade mathematics achievement. In the 2004 PISA study, the U.S. ranked 24th of 29 countries in the ability of 15-year-olds to solve real-world mathematics problems. The results suggest that, at the secondary-school level, the learning gap between the United States and its competitors in Europe and Asia is widening.

II. Defining and Navigating Intervention Solutions

Educational interventions provide teachers with the tools to deliver meaningful learning activities that improve academic performance and modify behavior for students who have already failed and need credit recovery, and for borderline students who require immediate support to avoid failure. All of these students can benefit from intensive, individualized attention.

In this report, we will help navigate the complex and under-defined educational category of **intervention solutions**. We discuss pedagogical approaches, including key components and descriptions of types of interventions in general as well as solutions related to special populations. Next, we illustrate how Carnegie Learning's Cognitive Tutor® math curricula fit with and exemplify these general findings and principles. We introduce research specific to math education and, finally, provide evidence that the Cognitive Tutor yields measurable results in an intervention context.

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III. Background on Educational Interventions

Key Components of Interventions

One consideration in all interventions is increasing **time on task**. It is unrealistic to expect students who are one or more grade levels behind to be able to “catch up” without increasing instructional time. One of the most well-established facts about how people learn is that learning takes time. Time is needed to practice, so that facts are easily remembered and procedures fluently executed. Time is also needed to integrate new learning into a student's existing understanding. We discuss some of the reasons why time is so important to learning below.

Increasing time on task is important, but it is equally important that students' time is spent well. In order to use students' time well, it is important to understand what students already know and to provide them with instruction, activities and experiences that are appropriate to their abilities and prior knowledge. Two of the most effective techniques in educational interventions are **formative assessment** and **differentiated instruction**.

- ▶ **Formative assessment:** Formative assessment is assessment for the purposes of understanding student knowledge so that instruction can be targeted exactly at the knowledge and skills that students need to learn. Formative assessment can be done at a classroom level, as when a teacher provides a pretest that identifies areas in which the class is weak. It can also occur, often less formally, on an individual basis, as when a teacher identifies and remediates a specific misconception that a student harbors.
- ▶ **Differentiated instruction:** Another key component of effective educational interventions is **accommodating individual differences** in student learning within a classroom. Differentiated instruction involves a range of techniques designed to assist student learning at all ability levels, and promote academic success by **reacting responsively** to a wide range student backgrounds and abilities. The goal in differentiated instruction is to provide each student with an individualized learning experience, within a group classroom environment. With the increasing diversity of the American classrooms (ranging from students with disabilities to gifted students), the demand has risen for differentiated instruction (Tomlinson and McTighe, 2006; Hall, 2002).

Types of Interventions

Proactive interventions are required for students who have not yet failed but who are judged either by grades or standardized assessments to be at risk of academic failure, or who are performing below grade level. These interventions are typically implemented before or at the beginning of a school year, either as a remedial course during *summer school* prior to the year in concern, or as a *dual-curriculum* schedule for a student, replacing study hall or elective course.

Credit recovery interventions, for students who have already failed, must begin immediately in an attempt to avoid further credit loss through extended time. Summer school is an option if intervention prior to the end of the school year is not realistic. Otherwise, remediation should begin in January or February of the school year in the following formats:

- 1. Pull-out programs** in which the student is removed from the course that was failed the fall semester, and placed in a remedial course which, by the end of the term plus summer school, allows the student to catch up with his or her peers.
- 2. After school hours** either at school or off site
- 3. Two-period course schedule (dual-curriculum)**

Overall, ensure the same instructional conditions under which the student has already failed are not repeated. That is, “more of the same” is not going to bring about success. Instead, a change of strategy in an attempt to reach students with a variety of learning styles is required.

Small learning communities. On a larger scale, interventions can be modeled as fundamental, **structural changes** to the design of a school schedule. Each of the models described in this section is promoted by the U.S. Department of Education as a structure or strategy for creating small learning communities. The Department of Education defines a Small Learning Community as an *“environment in which a core group of teachers and other adults within the school knows the needs, interests and aspirations of each student well, closely monitors each student’s progress, and provides the academic and other support each student needs to succeed”* (Department of Education, 2006).

Thus, students below grade level in math may benefit from a strategic intervention such that, after a year or two, they can “participate successfully in rigorous academic courses that will provide them with the educational foundation to enter college, apprenticeships, or other advanced training or employment” (Department of Education, 2006). Three models of small learning communities are illustrated below: the Academy, Alternative Scheduling, and Multi-Year Groups.

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- ▶ **Academies** establish a “school within a school” organized around a particular subject or theme that expands the options found in a traditional school curriculum. Some examples are: Technology Academy, Math (and Science) Academy, Health Sciences Academy, Visual and Performing Arts Academy. Such academies are designed to provide opportunities for students to focus on and foster interests and talents that fall outside mainstream school curricula.
- ▶ **Alternative Scheduling** is a strategy intended to provide opportunities for teachers to facilitate the alignment of lessons and learning objectives by creating longer study periods. This change may be made at the class period, school day or school year level, and is most easily accommodated in smaller schools. A common alternative that has been implemented for many years is the “**block schedule**”, extending class periods to give teachers time for deep, experiential learning activities as well as to create an environment conducive to differentiated learning (individual work with students who are behind, as well as more intensive lessons for advanced students).
- ▶ **Multi-year grouping** builds a smaller learning community by connecting students with a group of teachers over multiple years. This strategy, similar to “looping”, is designed to cultivate stronger relationships between student and teacher.

Adaptations for special populations. Interventions may also benefit special populations, with some modifications. For **special education** students, teachers must be sure to set realistic expectations for the students. For instance, Algebra I may need to be expanded into a two-year course. **ESL students** may require language support instructors or other additional assistance. Regardless, interventions must bear in mind that these student populations benefit tremendously from individualized instruction.

Key

	Recommended
	Appropriate
	Inappropriate









	Summer School	Dual Curriculum	After or out-of-school	“Pull-Out” Program
Proactive				
Credit Recovery				

Figure 1: Relative effectiveness of mathematics intervention models for Proactive and Credit Recovery programs.

IV. Why Carnegie Learning's Cognitive Tutor is a Proven Intervention Solution

Carnegie Learning's Cognitive Tutor curricula integrate formative assessment and differentiated instruction.

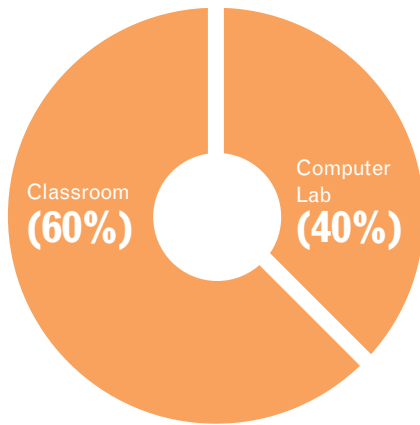
Two Key Components of Education Interventions

- ▶ **Formative assessment.** In any given lesson, there may be up to twenty skills for a student to learn. The Tutor constantly monitors the student's actions and each action is tied to the larger set of skills that comprise the domain, such as labeling a graph or finding negative slopes. To give meaningful feedback to the student, the Tutor estimates the probability that the student has learned each required skill and displays the results in a bar chart called the Skillometer. When students perform a correct action they can measure their progress on the chart, but if they make a mistake the bar chart shows a decrease. When remedial work is required, the Tutor picks problems that concentrate attention on the skills in which the student is weak. Students graduate to the next lesson after sufficiently demonstrating all required skills.
- ▶ **Differentiated instruction.** Another critical component of the Carnegie Learning curricula is that the students receive differentiated instruction. The Tutor checks every action performed by the student – entering a value, clicking a button, selecting a menu item – against the cognitive model. Effectively acting in the same way that a human tutor would, if the student action is inappropriate, the Tutor will not let the student go astray. Instead, it will guide the student to a correct solution path. The cognitive model recognizes that there are multiple ways to solve any particular problem and the Tutor restricts activities only when they would lead the student in the wrong direction.

Carnegie Learning's Pedagogy and Intervention

An increasing emphasis on accountability and testing, especially high-stakes graduation assessment, demonstrates an urgent need for intervention instruction including test preparation, remediation, and credit recovery. Current intervention methods often involve a random aggregation of existing materials that provide practice test items on practice exercises with some expository instruction for students, but no clear methodology for measuring student comprehension or assessing improvement. Implementing Carnegie Learning's Cognitive Tutor software provides a cost effective, easily deployed intervention curriculum solution to help struggling math students prepare for the future.

Figure 2: The standard implementation of all Carnegie Learning blended curricula is three days of textbook collaboration in the classroom and two days in the computer lab with the software.



Implementing Carnegie Learning's Curricula in Intervention

Carnegie Learning's math curricula capitalize on the benefits of these intervention models. **Summer school** can be implemented for both proactive and credit recovery interventions. A typical summer school program presents approximately 45 hours of curriculum (three weeks for three hours/day). On its own, Carnegie Learning's Cognitive Tutor software is designed to be completed in approximately 50 hours, which means a student could complete most of a course in a summer school term. Alternately, if the entire blended curriculum (60% textbook and 40% software) is used, the student would use the software for approximately 18 hours, and dedicate that time to mastering the weakest fundamental skills.

Pull-out programs. This type of intervention is available as a credit recovery intervention. When a student fails early in the school year or scored poorly on the previous year's standardized test and a passing grade in the course is improbable or impossible, a student may be "pulled out" of that course (either temporarily or for the remainder of the school year) and placed in a remedial course. The goal of this remedial course would be to recover the credit lost the previous term as well as to prepare the student for the next school year without falling behind.

Examples: A student fails the traditional Algebra I course in fall semester. For spring semester she is placed in a remedial class lasting through summer school, with the Carnegie Learning Algebra I curriculum, to not only remediate the previous semester credit loss, but also to master the required skills for that school year.

Another student scored poorly on the end-of-year 8th grade state exam, and is unlikely to do well in Algebra I in the upcoming school year. As a result, he is enrolled in summer school to work on the weaker mathematics skills with Carnegie Learning Bridge to Algebra, and stay on track to take Algebra I in 9th grade.

After school or out-of-school implementations are useful for both proactive and credit recovery interventions. Home computers may be the best after school option, but are not always available. Other possibilities are youth centers, after school programs at school, YMCAs, libraries and community centers. A remote access disk is available for any student.

Examples: Chicago checks out their remote access disks to students for installation at home or on other community computers. Students use the software outside of class, and teachers are able to monitor the progress made both in and out of school. And Marcia Andrus' students at Claremont HS (Claremont USD, CA) voluntarily enroll in a tutoring session every day after school for about a half an hour to spend additional time completing the software units and written work and assignments.

Dual-curriculum models. The standard instructional format for a single curriculum school year prescribes three days a week of classroom instruction and two days a week in a teacher-supervised software lab environment. A dual-curriculum model may be designed with concurrent or *double-period* classes. An implementation of a concurrent dual-curriculum design would have students in one school year taking Bridge to Algebra in one period and Algebra I in another, replacing study hall or other electives with the second math course. A *double-period* dual-curriculum design would schedule remedial students with Bridge to Algebra for two periods during the first semester, then Algebra I for two periods during the second semester. Both designs of the dual-curriculum model bring students closer to understanding and performing at grade-level.

Example: The State of Washington decided that no mathematics courses would be offered below grade level. Cascade HS (Everett SD) first used Bridge to Algebra for students who had failed Algebra I one or more times, as returning to the same curriculum was not an option. Until students reached a specified skill level, an elective was replaced with Bridge to Algebra while enrolled in Algebra I. Now, every high school in the district uses Bridge to Algebra to supplement Algebra I students. All middle schools in the district use Bridge to Algebra in conjunction with the traditional math curriculum in double periods. Eisenhower HS (Everett SD) works to align the learning objectives in both the Bridge to Algebra and traditional course.

Examples of Cognitive Tutor Implementations in Small Learning Community Models

Despite weaknesses in and criticisms of some Small Learning Community school implementations, we believe the underlying principles should be considered in a discussion of mathematics intervention solutions. Illustrations of sample implementations are provided below.

Math Academy. One type of smaller school structure is the academy, or a school sub-group based on a particular theme. Academies also emphasize building relationships between students and adults. We recommend creating a Math Academy based on the small learning community model, to integrate Carnegie Learning curricula-based academic instruction with vocational applications, providing learning opportunities for students, enhancing real-world relevance and creating a focused learning community.

Block Schedule. The school schedule may be only partially modified so that the overall arrangement is still six or seven periods with two periods allocated for math. Alternately, the entire schedule may be shifted to a 4x4 model, with

students studying four courses each semester, with 80-minute periods.

The Carnegie Learning curricula work with all variations and implementations of the block schedule model, and are designed to be used as a blended curriculum (both software and textbook). One example of a typical 80-minute period:

- ▶ The first 5 minutes: teacher gives overview of day, students log into computers
- ▶ The next 40 minutes: students work independently on the software
- ▶ The next 5 minutes: students take a break and shift into the classroom mode
- ▶ The last 30 minutes: the teacher spends on the day's lesson with lecture combined with collaborative learning in student groups

Multi-Year groups. Students are grouped by math ability level. These students stay together over the course of two or more school years, with the same set of teachers for all subjects. The teachers work together in planning lessons, with particular emphasis and attention to the objectives in the math curriculum.

Example: At The Madison School, the 9th grade class is subdivided into multi-year groups. Some students ("pink panthers") are slated for a dual-curriculum math schedule, with Bridge to Algebra remedial program in one period, and Algebra I in another. Students in another group ("purple people eaters") with more advanced math skills are in Algebra I only. A third group with a much wider range of skills ("red robins") have block schedules, and spend longer periods working on Algebra I (with access to Bridge to Algebra and Algebra II, as needed), allowing the teacher to plan for differentiated instruction and accommodate student needs at all ability levels.

TABLE 1 Sample 5-year model for implementing and evaluating mathematics intervention.

Grade in School	Year 1	Year 2	Year 3	Year 4	Year 5
Training, support and PD for all involved teachers and administrators for 8 consecutive years					
10th Grade	Algebra 1 (A1)	Follow 11th grade: enroll in adv. math	Follow 12th grade: grad rate	Enroll in college, apprentice, job	Performance out of HS
9th Grade	Bridge to Algebra (BTA)	A1	Follow 11th grade: enroll in adv. math	Follow 12th grade: grad rate	Enroll in college, apprentice, job
8th Grade		BTA	A1	Follow 11th grade: enroll in adv. math	Follow 12th grade: grad rate
7th Grade	(BTA)		BTA	Follow 11th grade: enroll in adv. math	Follow 11th grade: enroll in adv. math
6th Grade				BTA	A1
5th Grade					BTA

Early Intervention for Special Education

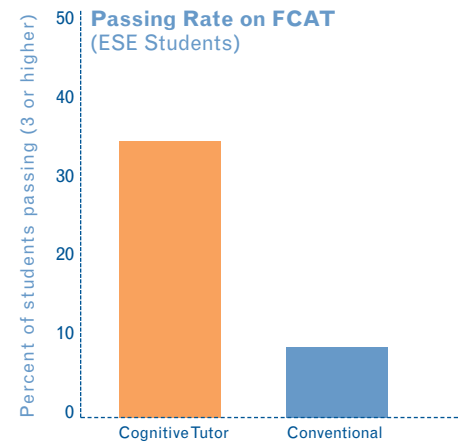
Many special education students are in mainstream-level classes. The individualized instruction provided by the Cognitive Tutor software allows students of all ability levels to progress at their own pace. In addition, the collaborative learning component of the textbook helps students at lower ability levels to learn from more advanced students.

The Carnegie Learning curricula are designed to meet the needs of all students, and we show consistently positive results with students who have been left behind in by traditional math instruction.

An independent study by the Reliability Group (Sarkis, 2004) found that ESE students who used Carnegie Learning's Cognitive Tutor Algebra I curriculum dramatically outscored similar students using traditional math curricula on Florida's FCAT exam. In fact, 35.7% of Cognitive Tutor ESE students passed the FCAT, as opposed to only 10.9% of comparison students.

Overall, interventions are designed and implemented with two primary goals: (1) to increase time on task, and (2) to provide a different approach to learning. Interventions are best understood in the context of how students learn, which is described in the next section.

Figure 3: Cognitive Tutor ESE students, whose FCAT scores were 44 points higher than those of ESE students using a conventional curriculum (Sarkis, 2004).



V. Research on How Students Learn

A basic goal in the learning sciences is to produce a model of how the mind works. Such a model explains basic facts about learning and performance, such as why we remember some things and forget others, how we solve problems and how we use language. The model explains the way that memories are stored in our brain and the way that perceptual information from our senses brings memories to our consciousness. A model that aims to explain the full range of human cognition is known as a Unified Theory of Cognition (Newell, 1973, 1990; Anderson and Lebiere, 2003). The most widely used and robust candidate for such a theory is known as ACT-R (c.f. Anderson, 1990, 1993; Anderson & Lebiere, 1998).

Overview of ACT-R

ACT-R is both a theory and a computer model. Implementing ACT-R in a computer requires the theory to be precise about all of its claims. It also allows Cognitive Science researchers around the world to simulate human behavior within an ACT-R model. In most cases, the researchers do this to understand the details underlying human behavior. If they can reproduce important characteristics of human behavior, including error patterns and response times, then they better

understand the details about how the mind works. Hundreds of researchers currently use ACT-R in exactly this way. Thousands of peer-reviewed publications have resulted (see <http://act-r.psy.cmu.edu/>).

A full explanation of ACT-R is beyond the scope of this paper. In this section, we discuss procedural and declarative knowledge, working memory and their implications for education.

Types of Knowledge

Knowledge comes in two types: declarative and procedural. Declarative knowledge includes facts, images and sounds. Procedural knowledge is an understanding of how to do things. All tasks involve a combination of the two types of knowledge. As we learn, we generally start out with declarative knowledge, which becomes proceduralized through practice. Procedural knowledge tends to be more fluent and automatic. Declarative knowledge tends to be more flexible and also more broadly applicable.

The interaction of declarative and procedural knowledge can be seen in learning to touch type on a keyboard. Initially, this task is largely driven by procedural knowledge. If you wish to type without looking at the keyboard, you must actively think about, for example, where the 'e' key is located. As you repeat this activity many times, the ability to strike the 'e' key becomes more automatic. A beginning typist may rely on a mix of procedural and declarative knowledge, sometimes hitting the key without thinking and sometimes struggling to visualize the keyboard. With practice, most people find that they can type rapidly and with few errors, and that they expend little mental effort in the act of typing.

At some point, many people find that, although they can type rapidly, they are not easily able to describe the location of particular keys on the keyboard. For such people, procedural skills have been strengthened but the declarative knowledge of the location of the keys is relatively weak. We do not believe that the procedural knowledge replaces the declarative knowledge. More likely, procedural skills continue to be strengthened through practice when we type, but the declarative knowledge of talking about or visualizing particular keys becomes weak through disuse.

Both declarative and procedural knowledge become strengthened with use (and weakened with disuse). Strong knowledge can be remembered and called to attention rapidly and with some certainty. Weak knowledge may be slow, effortful or impossible to retrieve. Furthermore, the time course of knowledge use affects its resistance to forgetting. Although "cramming" for a test by studying intensely for a short period of time will help in the short term, long-term memory is best served by appropriately spacing study time. This outcome is likely because some of the

context in which the learning event takes place is encoded with the knowledge itself, so spacing practice over a period of time helps the fact or procedure to gain multiple contexts in encoding (c.f., Madigan, 1969; Gay, 1973; Glenberg, Smith & Green, 1977).

It is important to remember that the mind can not distinguish correct from incorrect knowledge. In the absence of feedback from the world (or internal reflection about whether new knowledge fits with what you already know), there is nothing to prevent incorrect knowledge from being strengthened. “Two plus two equals five” is a perfectly fine piece of declarative knowledge, and someone who practices that knowledge will strengthen it.

Applications to Education

Because of the distinct characteristics of procedural and declarative knowledge, robust understanding of a domain involves both types of knowledge (Rittle-Johnson and Koedinger, 2001; Koedinger and Terao, 2002; Siegler, 2002). Procedural knowledge is fast and requires little mental effort. This allows more of our mental energy to be focused on new learning or more complex or novel tasks. However, procedural knowledge can also be fragile and context-specific. In the event that some procedural knowledge is forgotten, declarative knowledge can be used to reconstruct the procedural knowledge. Declarative knowledge can be reasoned about in a way that is not possible with procedural knowledge. This makes declarative knowledge more flexible and more easily applied in a new context. For example, a student who has learned basic algebraic transformations (e.g. add, subtract, multiply, divide) may use declarative knowledge to reason about whether taking the square root of both sides of an equation is an action that preserves the equality.

Knowledge is strengthened through practice, and strong knowledge is more likely to be available when needed. However, it is possible to strengthen both correct and incorrect knowledge.

The Structure of Procedural Knowledge

The previous discussion of types of knowledge may give the impression that procedural knowledge is the same as declarative knowledge, except that it is stored in a form that can be rapidly executed with little mental effort. In fact, the structure of procedural knowledge is more complex.

Procedural knowledge consists of a large set of if-then rules. The “if” (or condition) part of the rule specifies a condition under which the rule may be applied. The condition often specifies the user’s goal as well as some aspects of the context.

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The “then” (or action) part of the rule specifies some change to make as a result of that rule. The action may dictate a change in mental state (like new declarative knowledge) or it may specify an action (like speaking a response). As an example, one rule might be:

IF
the goal is to solve an equation for X
and
the equation is of the form $aX=b$
THEN
divide both sides of the equation by a

Note that this rule only suggests a possibility, not a requirement. Someone who has the above rule might also have the similar rule:

IF
the goal is to solve an equation for X
and
the equation is of the form $aX=b$
THEN
multiply both sides of the equation by $1/a$

Both rules contain the same conditions, so either may be used when the conditions apply. Which one is actually used depends (probabilistically) on the strength of the rule (which, in turn, depends on the rule’s past use). Use of one rule or another constitutes use of different solution strategies.

Note that the procedural knowledge presented here is greatly simplified for explanation purposes. Either of these rules could be part of a sequence of mental actions. Execution of one of these rules would result in a mental state where the goal might be “divide both sides by a” and a number of rules could be executed to implement that goal.

Mental rules may also incorrectly encode features of the environment, resulting in fragile knowledge (Alevan, Koedinger, Sinclair and Snyder, 1998). For example, a student may encode a rule like:

IF
the goal is to solve an equation for X
and
the equation consists of a term equal to a constant
and
that term contains a number in front of X
THEN
divide both sides of the equation by the number

This form of the rule relies on a surface feature of the equation, that there is a number in front of X , rather than an understanding of that mathematical meaning (that the number multiplies X). For most equations, this distinction does not matter, but, for an equation where the coefficient of X is -1 , the rule does not apply. Thus, if a student has learned a rule of this form, the student may not be able to solve equations of the form $-X=a$.

Applications to Education

Procedural knowledge consists of a number of different rules for solving problems. These rules function as possible strategies or approaches to a problem. Students simultaneously possess multiple approaches to a problem (both correct and incorrect). Through practice, correct and efficient procedures can be strengthened.

Since a number of different procedural rules may implement a single surface behavior, it is important that students practice procedures in a number of different contexts and with problems containing varying characteristics.

The Structure of Declarative Knowledge

While we have previously described declarative knowledge as being a collection of facts (including images, sounds, etc.), these facts are not stored independently, as in a list. Rather, facts are highly interconnected. Along with the facts themselves, the mind stores relationships between facts (c.f. Bower, Clark, Lesgold and Winzenz, 1969; Meyer and Schvaneveldt, 1971).

Facts are also not just static entities. They can be “activated,” meaning that they are brought into active consideration. Activation determines how easily a fact can be remembered and used. One way to think about consciousness and awareness is that we are aware of the most highly activated parts of our memory.

The relationship between the interconnections in declarative knowledge and activation is crucial to the way that memory works. Thinking about one fact (e.g. “dog”) causes some activation of related facts (e.g. “animal” or “cat”). This “spreading activation” allows us to focus on related topics. Retrieving memories involves starting with a focus of attention (perhaps from a cue in the environment) and spreading that focus to remaining items, which can be examined to see if they are the target of the memory retrieval. For example, the cues “president” and “Civil War” may lead to activations for “Abraham Lincoln” and also to “Ulysses S. Grant.”

The better connected a fact is to other facts and concepts in memory, the better a student will be able to retrieve that fact. Memorizing a list of American presidents may establish a link between “president” and “Ulysses S. Grant” but, without providing some of Grant’s biography (such as his role in the Civil War), the fact will not be well connected to other facts, other than the list of presidents.

Applications to Education

When facts are learned, they are placed into a highly interconnected network of declarative memories. If the fact is to be remembered, it must be related, directly or indirectly, to the retrieval cues. Well-rehearsed facts will be strengthened and thus more easily activated, so they can be retrieved more easily.

Because of the interconnectedness of memory, it is important that facts and concepts be learned in a way that can be related to prior knowledge. The better connected a fact is to other facts and concepts in memory, the better a student will be able to retrieve that fact. Memorizing a list of American presidents may establish a link between “president” and “Ulysses S. Grant” but, without providing some of Grant’s biography (such as his role in the Civil War), the fact will not be well connected to other facts, other than the list of presidents.

Connections between new information and prior knowledge will be more easily established when the new material fits with the student’s prior knowledge and when connections with prior knowledge are highlighted (Reder and Anderson, 1980; McNamara, Kintsch, Songer, & Kintsch, 1996; Kintsch, 1988).

In addition, learners who self-explain and elaborate on textbook materials (relating the new material to material learned earlier) form deeper connections with their existing knowledge, leading to more robust learning (Chi, Bassok, Lewis, Reimann & Glaser, 1989; Chi, de Leeuw, Chiu, & LaVancher, 1994). Finally, there is evidence that students who provide explanations for numerical conclusions exhibit more robust learning (Alevan & Koedinger, 2002).

Working Memory Load

While our memory consists of a large collection of declarative facts and rule-based procedural knowledge, most of this knowledge is inert. That is, the knowledge is not immediately available to us to be used. In order to use knowledge, it must be activated (which may not reach the level of consciousness) and then used. One of the basic limitations on our ability to think is a limit on the amount of information that can be activated at one time. Active information is called “working memory.”

One way to think about working memory is that it is a limit on the number of “things” that you can think about, and that limit is often described as about seven (Miller, 1956). However, this kind of description brings up the question of what counts as a “thing.” Is it a number or is it each of the digits in the number? The answer appears to be related to the organization of memory. Things that are closely related into a “chunk” (like the digits within your phone number) effectively act as a single “thing.”

The strain on working memory (often called “working memory load”) is also related to the strength of a memory (stronger knowledge requires less effort to maintain in working memory) and to the extent to which processes are proceduralized. The reason why highly practiced procedures (like touch typing) appear to require little mental effort is that the procedural skills take up little working memory, which frees your mind to think about other things.

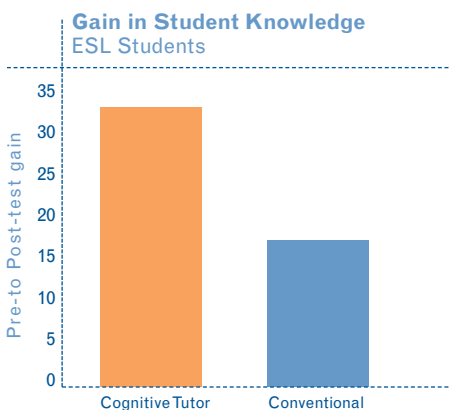
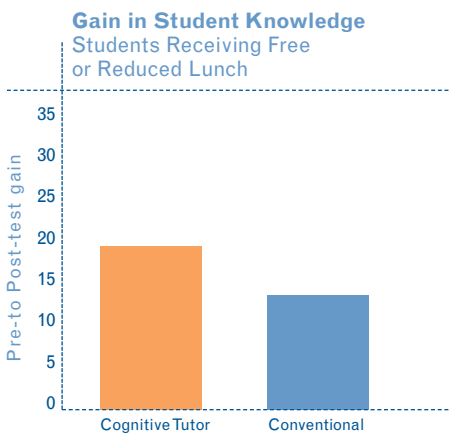
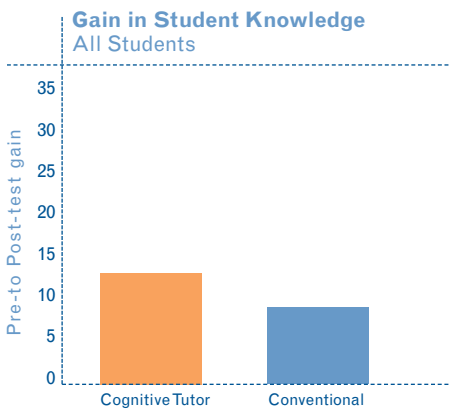
Applications to Education

New information, which is, by definition, not well strengthened, will take up a relatively large amount of working memory. For this reason, it is best to reduce the working memory load within educational tasks, especially when the load is not related to the target of instruction (Sweller, Chandler, Tierney and Cooper, 1990). For example, when teaching equation solving, the use of small integers as coefficients reduces the working memory load associated with remembering and performing arithmetic on those coefficients, freeing up working memory for learning the rules relating to algebraic transformations (Anderson, Reder and Lebiere, 1996).

Relating new information to old may promote the establishment of memory “chunks,” which will reduce the student's working memory load. In general, building on prior knowledge will tend to activate connected memory items, reducing memory load at learning time.

Intervention materials must be carefully selected, keeping in mind the pedagogical designs that build upon student knowledge and heed the fundamental principles of learning. Carnegie Learning's Cognitive Tutors were developed at Carnegie Mellon University as part of a research project by **world-renowned cognitive scientists** who were testing a theory of how people learn, and are **based on over 20 years of research** into how students think, learn and apply new knowledge in mathematics. The Tutors use students' intuitive problem solving abilities as a powerful bridge to more formal and sophisticated mathematical comprehension. Our research references demonstrate that the Cognitive Tutors are not only based on solid learning science, but also that our publications have appeared in respected, peer-reviewed professional journals.

Figure 4: Gary Plano in Kent SD (WA) used a regression discontinuity design. 1,027 9th grade students participated. Effects were most pronounced for ESL and FRL student populations (Carnegie Learning, 2004).



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VI. Evidence that Carnegie Learning's Cognitive Tutor Works

All of the Cognitive Tutor mathematics curricula from Carnegie Learning are based on extensive scientific research from Carnegie Mellon University, along with field tests in schools throughout the United States. The Cognitive Tutors are based on the ACT-R theory of learning, memory and performance, which has been validated by hundreds of lab and field studies. The Tutors themselves were developed using a rigorous empirical testing process resulting in over 50 publications validating the effectiveness of cognitive modeling.

For example, a study conducted in Kent, Washington School District demonstrated a pre- to post-test improvement on the NWEA's Achievement Levels Test (ALT). English Language Learners who used Carnegie Learning improved 31 points from pre- to post-test, as opposed to a 17-point improvement for students using a traditional curriculum. The study also showed that students receiving free or reduced lunch improved 19 points if they used Carnegie Learning, as opposed to 14 points using a traditional curriculum.

Our Results at a Glance

Carnegie Learning's Cognitive Tutors support improved student achievement in mathematics. To keep our programs current, and to make certain the teachers using our curricula have the benefit of thorough research, we continue to support investigations into the efficacy of our programs.

Research has shown that students using the Cognitive Tutor Algebra I program:

- ▶ Perform 30% better on questions from the TIMSS assessment
- ▶ Demonstrate an 85% better performance on assessments of complex mathematical problem solving and thinking
- ▶ Have a 70% greater likelihood of completing subsequent Geometry and Algebra II courses
- ▶ Achieve 15-25% better scores on the SAT and Iowa Algebra Aptitude Test
- ▶ Experience equivalent results for both minority and non-minority students

For more details, see http://www.carnegielearning.com/web_docs/cmu_research_results.pdf

Current Research

Carnegie Learning's current research activities include:

- ▶ Large-scale randomized field evaluation of Cognitive Tutor Geometry
- ▶ Randomized field evaluation of Cognitive Tutor Algebra I in four school districts
- ▶ Over 20 controlled experiments on variations of the Cognitive Tutors in conjunction with the Pittsburgh Science of Learning Center
- ▶ Analysis of student learning in Bridge to Algebra, using a data set that represents the most detailed record of student mathematical behavior ever collected
- ▶ Working with Southern University, a historically Black college in Baton Rouge, Louisiana, to build tools allowing teachers to build their own Cognitive Tutor activities
- ▶ Working with Carnegie Mellon University and Worcester Polytechnic Institute to develop statistical methods for using data from Cognitive Tutors to predict and improve scores on state tests

VII. Conclusion

Over the past several decades, concern with high dropout rates and adult illiteracy resulted in initiatives that have proven successful in improving reading programs and literacy across the country. A similar awareness of the need for math and science education reform has emerged. This time, however, educators are not alone in creating awareness. The economics of America's declining global competitiveness and the pervasiveness of off-shore outsourcing of technical talent, fueled by the interests of large U.S.-based corporations, is driving high reform initiatives nationwide.

Mathematics is at the foundation of a science or engineering degree, and a solid understanding of mathematical concepts and principles will provide our students with the tools to achieve, succeed, and compete in the global marketplace. Federal and state-sponsored programs such as the President's American Competitiveness Initiative announced in February 2006 and the National Governors Associations Innovation America program unveiled in August 2006 are raising national awareness for the reform of math and science curricula and teaching strategies. The objective of these programs and many other initiatives sponsored by the Bill & Melinda Gates Foundation, the GE Foundation, and other non-profits, is to graduate a larger percentage of students well equipped to fill higher-paying, technical position in the years ahead.

The economics of America's declining global competitiveness and the pervasiveness of off-shore outsourcing of technical talent, fueled by the interests of large U.S.-based corporations, is driving high reform initiatives nationwide.

In August 2006 the National Science Foundation released survey results pointing to the benefits of earning a bachelor's degree in science or engineering (S&E) in the workforce, regardless of career path.

“Some 400,000 sales workers, for example, reported their job was related to their S&E bachelor's degree. And a majority of S&E bachelor's degree holders employed as artists, editors or writers reported their degree was at least somewhat related to their job.

Of those who went on to receive advanced degrees, the largest proportion, almost 29 percent, took those degrees in non-S&E fields, namely business, law or medicine.”

We believe that intervention programs in school districts across America will be the recipients of new funding, manpower, and other key resources in an effort to achieve global competitiveness. While math intervention strategies remain ambiguous and undefined in many schools, they will not for long. Carnegie Learning looks forward to providing guidance, curricula, and professional development to help districts navigate the many intervention options and to find the optimal, customized solution as we work together to improve the futures of our greatest resource – our children.

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About Carnegie Learning

Carnegie Learning is a leading developer of core, full-year mathematics programs as well as supplemental intervention applications for middle school and high school students. The company's Cognitive Tutor® is helping more than 375,000 students in more than 1000 school districts across the United States succeed in math by integrating interactive software sessions, text, and student-centered classroom lessons into a unique learning platform for Bridge to Algebra, Algebra I, Geometry, Algebra II and Integrated Math programs. The U.S. Department of Education recognizes Carnegie Learning's Cognitive Tutor Algebra I program as one of the only math curricula scientifically proven to have significant, positive effects on student learning. Based in Pittsburgh, PA, Carnegie Learning was founded by cognitive science researchers from Carnegie Mellon University in conjunction with veteran mathematics teachers. For more information visit (www.carnegielearning.com).

Christy L. McGuire, Research Manager

Christy McGuire is committed to lifelong learning. She has six years of experience in the education industry addressing age groups from adolescents to older adults. McGuire has served as a tutor for at-risk middle school students, as adjunct faculty member at Chatham College in Pittsburgh, PA, and as Area Director for Elderhostel, a non-profit organization educating healthy older adults. She earned a Ph.D. in Experimental Psychology at Georgia Institute of Technology in 2001. McGuire's primary area of expertise is in metacognitive components of learning and memory.

Steve Ritter, Chief Product Architect

Steve Ritter, Chief Product Architect at Carnegie Learning, has been developing and evaluating educational systems for over 10 years. He earned his Ph.D. in Cognitive Psychology at Carnegie Mellon University in 1992 and helped to found Carnegie Learning in 1998. As a postdoctoral associate and research scientist at Carnegie Mellon, Dr. Ritter was instrumental in the development and evaluation of the Cognitive Tutors for mathematics. He is the author of numerous papers on the design, architecture and evaluation of Intelligent Tutoring Systems.



Frick Building
20th Floor
437 Grant Street
Pittsburgh, PA 15219

Phone (Toll Free): 888.851.7094
Local: 412.690.6284
Fax: 412.690.2444
info@carnegielearning.com
www.carnegielearning.com

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